# Economics 2450A: Public Economics and Fiscal Policy I

Section 9: Generalized Social Welfare Weights

Michael Droste

Fall 2022

# Outline

- 1. Generalized Marginal Social Welfare Weights (Saez and Stantcheva 2016)
  - Motivation / Review
  - Individual Characteristics
  - Optimal Tax Formulae
  - Optimal Nonlinear Tax with Fixed Incomes

### **Review: Marginal Social Welfare Weights**

- From the first few weeks of class: optimal tax formulas depended on functions of marginal social welfare weights g<sub>i</sub>, which were defined in terms of an underlying (welfarist) social welfare function.
- For instance, consider the social welfare function:

$$SWF = \int_i G(u^i) di$$

with  $u^i$  the realized (indirect) utility of household *i* and  $G(\cdot)$  an increasing, weakly concave, differentiable function, we can define the marginal social welfare weight as:

$$g_i = G'(u^i) \cdot u^i_c$$

# Generalizing the Marginal Social Welfare Weights

- This week's lecture and section follow Saez and Stantcheva (AER, 2016). This paper is super readable, and should be a good reference for you if any of this material needs further clarification. I also promise it's a fun paper to read. Read this paper!
- The objective of Saez and Stantcheva is to think about a broader class of  $g_i$ 's in our optimal tax formulae that are potentially not defined in terms of the social welfare function. These 'generalized' weights will behave exactly like the  $g_i$ 's we have seen before, and appear in our tax formulae in the same way.
- Generalized social welfare weights also exhibit some nice theoretical properties, like local Pareto optimality (review: what do you think this means?) when non-negative for all *i*.

### Saez and Stantcheva (2016): Setup

- Unit mass of households indexed by *i* maximize utility functions of the form:

$$U_i = U(C_i - V(Z_i; X_i^U, X_i^D))$$

where  $c_i$  is household *i*'s consumption,  $z_i$  is household *i*'s earnings, and  $x_i^u$ ,  $x_i^b$  are sets of individual-specific characteristics (more below).

- Functions *u* and *v* are common to all individuals; *u* assumed increasing and concave, *v* assumed increasing and convex, both differentiable everywhere.
- Characteristics *x<sup>u</sup>* enter only in utility function, not in social welfare weights; characteristics *x<sup>b</sup>* enter in both utility and social welfare weights.

### Saez and Stantcheva (2016): Setup

- Define the generalized social marginal welfare weight as:

 $g_i = g(c_i, z_i; x_i^{\mathcal{S}}, x_i^{\mathcal{D}})$ 

where *c* and *z* are consumption and earnings,  $x_i^s$  represents individual-specific characteristics that only impact the social welfare weight (does not appear in previous slide!) and  $x_i^b$  are individual-specific characteristics that impact both the social welfare weight and utility.

## **Individual Characteristics**

- We have three sets of individual-specific characteristics:  $x_i^s$ ,  $x_i^u$ ,  $x_i^b$ . It's worth reminding ourselves what each of these must satisfy:
- $x_i^U$ : characteristics that impact utility, but not social welfare weights.
- $x_i^b$ : characteristics that impact both utility and social welfare weights
- $x_i^s$ : characteristics that impact social welfare weights, but not utility
- Naturally, we do not need to worry about characteristics that neither impact utility nor social welfare weights. Otherwise, these definitions form a partition over individual characteristics.

## Individual Characteristics: Interpretation

- The individual-specific characteristics  $x_i^s$  and  $x_i^b$  enter in the definition of the generalized social marginal welfare weight  $g_i$ . Any characteristic in either of these is implicitly something that the government values for the purposes of redistribution through taxes.
- The individual-specific characteristics  $x_i^u$  are in the utility function, but do not enter in the social welfare function. Not fair game for redistribution through taxes.
- All of these characteristics may either be observed by the government or not. If they are unobserved and enter in *g<sub>i</sub>* (s or *b*), then we must 'aggregate up' to things the tax system can depend on (income and potentially observable characteristics)

#### Individual Characteristics and Redistribution



FIGURE 1. GENERALIZED SOCIAL WELFARE WEIGHTS APPROACH

*Notes:* This figure depicts the three sets of individual characteristics  $x^{b}$ ,  $x^{u}$ , and  $x^{s}$ . Characteristics  $x^{u}$  enter solely the utility function (i.e., they affect individual utilities and choices). Characteristics  $x^{s}$  enter solely the generalized social welfare weights (i.e., they affect how society values marginal transfers to each individual). Characteristics  $x^{b}$  enter both the utility function and social weights.

## Individual Characteristics: Aggregation

- Suppose that  $x_i^b$  includes, for instance, height (like the Mankiw paper about Talls vs. Shorts we discussed for tagging/commodity taxation.
- If the government can observe height: construct average social welfare weights by aggregating at each  $(z, x^b)$
- If the government cannot observe height (just earnings), construct average social welfare weights by aggregating at each *z* instead.

### **Optimal Taxes with Generalized Welfare Weights**

- All of our classic Saez-like optimal income tax formulas (linear; top linear; nonlinear) go through even when our social welfare weights *g<sub>i</sub>* are generalized (and so not generally derived from an explicit SWF).
- Top linear tax satisfies:

$$au = rac{1-ar{g}}{1-ar{g}+e}$$
 with  $ar{g} = rac{\int_i g_i z_i di}{\int_j g_i di \cdot \int_i z_i di}$ 

- Top nonlinear tax satisfies:

$$T'(z) = \frac{1 - \bar{G}(z)}{1 - \bar{G}(z) + \alpha(z) \cdot e(z)} \quad \text{with} \quad \bar{G}(z) = \frac{\int_{i: z_i \ge z} g_i di}{\Pr(z_i \ge z) \cdot \int_j g_i di}$$

- Local proofs (perturbation arguments) follow exactly the same as before.

## **Generalized Welfare Weights with Fixed Incomes**

- What does this approach buy us, apart from the ability to perhaps condition taxes on other observable characteristics? Leading example: optimal taxes with fixed incomes (no behavioral responses):  $z_i = z$  for all *i*.
- Recall from first week: if government is choosing nonlinear tax T(z) to maximize standard welfarist SWF, optimal T(z) implies consumption is constant across *i* (why?).
- Three issues with the old standard approach:
  - 1. A priori, complete redistribution seems very strong
  - 2. Sensitive to utility specification: optimal tax changes a lot for linear utility vs. utility with very slight concavity
  - 3. Can't handle heterogeneity in utility very well

#### **Generalized Welfare Weights with Fixed Incomes**

- Generalized social welfare weights provide a different lens to this problem!
- Let  $g_i = g(c_i, z_i) = \tilde{g}(c_i, z_i c_i)$  with  $\tilde{g}_c \leq 0$ ,  $\tilde{g}_{z-c} \geq 0$ .
- Two extreme / polar cases to consider:
  - 1. Utilitarian weights:  $g_i = g(c_i, z_i) = \tilde{g}(c_i)$  for all  $z_i$  with  $\tilde{g}(\cdot)$  decreasing.
  - 2. Libertarian weights:  $g_i = g(c_i, z_i) = \tilde{g}(z_i c_i)$  with  $\tilde{g}(\cdot)$  increasing.
- Optimal nonlinear tax w/ fixed incomes satisfies (for any z):

$$T'(z) = rac{1}{1 - \tilde{g}_{z-c}/\tilde{g}_c}$$
 so that  $0 \le T'(z) \le 1$ 

- Utilitarian case: T'(z) = 1. Libertarian case: T'(z) = 0. (why? take limits)

## **Generalized Welfare Weights with Fixed Incomes**

- Intuition: in this case, when the optimal tax is conditioned only on z (i.e. no observed individual characteristics  $x^b$ ,  $x^s$ ):
- Weights depend negatively on *c*: standard welfarist logic, a dollar is worth more in marginal utility terms for the poor
- Weights depend positively on z c: captures idea that those who pay more taxes (z c) more deserving of benefits/transfers
- At an optimum,  $g_i$  constant across z. Generalizes to case with more conditioning variables.

## **Eliciting Social Preferences**

- So, we can throw in arbitrary kinds of social preferences inside our marginal social welfare weights  $g_i$ , and all of the math will go through pretty much the same as before.
- This begs the question: how can we discipline our generalized marginal social welfare weights?
- One possibility is to use surveys to ask people how deserving of a given tax break a person with a given level of income and tax burden ought to be. Can use this information to calibrate  $\bar{g}$ .
- Question always emerges with surveys: are we eliciting true beliefs? Talk is cheap! Good experimental and survey work take these concerns very seriously.