Economics 2450A: Public Economics and Fiscal Policy I

Section 4: Optimal Income Taxation III (Mirrlees)

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Fall 2022

[a] These slides are based on materials passed down by previous Ec2450A teaching fellows. All errors are mine.

Outline

1. Optimal Nonlinear Tax (Saez 2001)
   - Review of Baseline Results (No Income Effects)
   - Overview of Extensions
2. Optimal Nonlinear Tax (Mirrlees 1971)
   - Setup
   - Ugly Government Problem
   - Local ICs
   - Nicer Government Problem
   - Solving with Optimal Control

Review: Optimal Nonlinear Tax (Saez 2001)

Optimal Nonlinear Tax (Saez): Setup

- Setup should look familiar by now!

- **Households** maximize heterogeneous utility functions $u^i(c,z)$, face budget constraint $c = z - T(z) + R$. For now, assume utility fn. is quasilinear in c (no income effects).

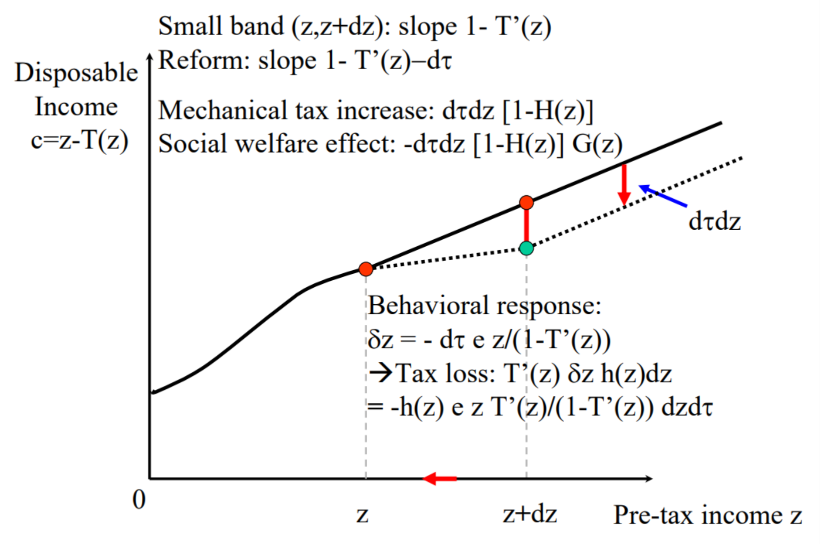
- **Government**: chooses tax schedule $T(z)$ to maximize generalized social welfare function subject to optimal household behavior.

- **To derive optimal tax**:
  1. Start with arbitrary tax schedule $T(z)$ and fix arbitrary z.
  2. Perturb marginal tax rate $T'(z)$ by a small amount $d\tau$ in small income band $[z, z + dz]$.
  3. Compute mechanical (dM), welfare (dW), and behavioral (dB) responses to perturbation.
  4. At an optimum $T(z)$, $dM + dW + dB = 0$. Impose and solve for $T'(z)$.

Optimal Nonlinear Tax (Saez): Setup

- A bit of useful notation: Let $h(z)$ denote the pdf of the income distribution and $H(z)$ denote the CDF of the income distribution (both endogenous).
- Mechanical impact: $dM = d\tau \cdot dz \cdot [1 - H(z)]$
  Review: Interpret each term. Why does $1 - H(z)$ appear?
- Welfare impact: $dW = -dM \cdot G(z) = -d\tau \cdot dz \cdot [1 - H(z)] \cdot G(z)$
  Review: Why is this term negative?
- Behavioral response: $dB = h(z) \cdot dz \cdot T'(z) \cdot \left[d\tau \cdot dz/d(T'(z))\right]$
  Review: Interpret each term, then write in terms of $e(z) = \frac{dz}{d(1-T'(z))} \cdot \frac{1-T'(z)}{z}$.
  Why does $h(z) \cdot dz$ enter in this term rather than $1 - H(z)$?

Optimal Nonlinear Tax (Saez): Key Figure



Optimal Nonlinear Tax (Saez): Optimal T'

- Next, define a *local* Pareto parameter at z to be $a(z) = zh(z)/1 - H(z)$. Plays the same role as a did in top linear tax rate model!

- Also define $e(z) = \frac{\partial z}{\partial(1-T'(z))} \cdot \frac{1-T'(z)}{z}$ as the local taxable income elasticity at z. Likewise, this is analogous to our e from before, but now the marginal tax rate varies with z.

- Setting $dM + dW + dB = 0$ and solving for T' yields the optimal (nonlinear) marginal tax rate for any given z:

$$T'(z) = \frac{1 - G(z)}{1 - G(z) + a(z) \cdot e(z)}$$

- Very nice: everything generalizes! The optimal marginal tax rate at any income level z depends on the Pareto parameter $a(z)$, the taxable income elasticity $e(z)$, and the average social marginal value of consumption for those with incomes above z, $G(z)$.

Optimal Nonlinear Tax (Saez): Retrospective

- Excellent question last week from Coly (paraphrasing): "Are these models/results still important in frontier research?" I was not clear in last week's section: the answer to this excellent question is an emphatic yes.

- The Saez approach's biggest virtue is that it is *easy to take to data*. All of last week's optimal income tax formulas did not depend on functional form assumptions for utility or model primitives; boiled down to e, Pareto parameter a (for top linear or nonlinear optimal tax), and $\bar{g}$.

- This makes it attractive for researchers aiming to write empirical papers - if you can estimate the taxable income elasticity, you can give it a theoretical interpretation in terms of an optimal tax problem.

- We will see that this starkly contrasts with the Mirrlees model today.

Optimal Nonlinear Tax (Saez): Extensions

- Several important extensions to this model to think about:
  1. Income effects: how should this change the impact of a tax? Think about dB.
  2. Migration responses: Fun, useful extension that drills home a good application of the envelope theorem to discrete choice.
  3. Tax evasion/avoidance.

- In the interest of time this week, I will not cover these directly. They don't involve any novel math or big new concepts - just small extensions of the baseline Saez framework. The Mirrlees framework is harder conceptually and the marginal value of covering that today exceeds the marginal value of covering the Saez extensions.

- Still, these will be important for the exam - make sure you understand the derivations and key take-aways!

Optimal Nonlinear Tax (Mirrlees 1971)

Mirrlees: Household Setup

- Households choose consumption c and labor supply ℓ to maximize utility subject to budget constraint. For now, assume utility function is quasi-linear in consumption (no income effects), so $u(c, \ell) = c - v(\ell)$ for concave v.

- Households are heterogeneous in exogenous ability n. Let f denote the density of the ability distribution and let F denote the cumulative distribution. For convenience, assume non-negative and unbounded support: $n \in [0, \infty)$.

- Household's labor income is $z = n\ell$; that is, each unit of labor ℓ allows the household of type n to purchase n units of consumption (real terms).

Mirrlees: Labor Supply and Labor Wedge

- Household utility maximization problem can be expressed in terms of z after subbing in budget constraint into utility function:

$$\max_z \; z - T(z) - v(z/n) \quad \implies \text{FOC:} \quad T'(z) = 1 - v'(\ell)/n$$

- Review: How can we interpret $v'(\ell)/n$? (Hint: think about a marginal rate of substitution)

- Observe that absent taxes, the FOC becomes $v'(\ell)/n = 1$. The wedge or distortion that taxes induce in labor supply choice captured by $T'(z)$.

- Totally differentiating FOC w.r.t. the after-tax wage $(1 - T'(z))n$ yields:

$$\frac{d\ell}{d\big((1 - T'(z))n\big)} = \frac{1}{v''(\ell)}$$

- $\implies$ elasticity of labor supply w.r.t. net-of-tax wage is $\epsilon = \frac{(1-T'(z))n)}{\ell \cdot v''(\ell)}$

Mirrlees: Mechanism Design

- The wrinkle here for optimal tax is that we will assume the tax is a function of income z, and not ability n. In this model, we've assumed households are ex ante heterogeneous in terms of ability. This induces ex post heterogeneity in terms of earnings.

- Review: What would happen in this model if the government observed ability n for each agent and wanted to maximize utilitarian social welfare with an ability tax $T(n)$?

- **The Revelation Principle**: If an allocation can be implemented through some mechanism, it can also be implemented through a *direct truthful mechanism* where the agents reveal their private information about n.

- Imagine that households *report* their type n', and allocations are a function of n'.

Mirrlees: Government Problem (Ugly)

- The government's problem is to *choose the allocations* $c_{n'}, z_{n'}$. This is equivalent to choosing the tax.
- Government maximizes social welfare subject to incentive compatibility constraints and resource constraint (with exogenous revenue requirement E):

$$\max_{c_n, u_n, z_n} \int_0^\infty G(u_n) f(n) dn \qquad \text{subject to}$$

$$c_n - v(z_n/n) \geq c_{n'} - v(z_{n'}/n) \quad \forall n, n' \qquad \text{(incentive compatibility)}$$

$$\int_0^\infty c_n f(n) dn \leq \int_0^\infty z_n f(n) dn - E \qquad \text{(resource constraint)}$$

- Ugly problem: huge number of IC constraints, one for each (n, n').

Mirrlees: Toward a Nicer Government Problem

- Fortunately, there are regularity conditions that will allow us to replace those IC constraints with a smaller set of IC constraints.

- The Spence-Mirrlees (or single-crossing) condition:

$$-\text{MRS} = \frac{v'(\ell)}{n \times u'(c_n)} \quad \text{decreasing in } n$$

- Strict monotonicity of allocations:

$$c_n', z_n' > 0$$

- When these conditions hold, *local* IC constraints are sufficient conditions for the problem. Proof of this is *way* beyond this course.

Mirrlees: Local Incentive Constraints

- When reporting their type n' to the government, a type n household is solving:

$$\max_{n'} \; u_{n'} = c_{n'} - v\left(\frac{z_{n'}}{n}\right) \quad \Longrightarrow \quad \text{FOC:} \quad c'_{n'} - \frac{z'_{n'}}{n} v'\left(\frac{z_{n'}}{n}\right) = 0$$

- Under truth-telling, $n' = n$, the FOC becomes $c'_n - \frac{z'_n}{n} v'\left(\frac{z_n}{n}\right) = 0$.

- Differentiating utility u_n wrt n, imposing FOC, substituting $\ell'_n = z'_n/n$:

$$\frac{du_n}{dn} = \left(c'_n - \frac{z'_n}{n} v'\left(\frac{z_n}{n}\right)\right) + \frac{z_n}{n^2} v'\left(\frac{z_n}{n}\right) = \frac{z_n}{n^2} v'\left(\frac{z_n}{n}\right) = \frac{\ell_n}{n} v'(\ell_n) > 0$$

  captures how utility changes with respect to type n. Convex v implies always positive.

- Informational rents: higher utility for higher types at an optimum. Higher n implies lower marginal disutility of labor for any given labor supply ℓ.

Mirrlees: Government Problem (Nice)

- Under Mirrlees-Spence single-crossing condition and monotonicity of allocations, can replace the continuum of incentive compatibility constraints with $\frac{du_n}{dn} = \frac{z_n}{n^2} v'\left(\frac{z_n}{n}\right)$, a "local" incentive compatibility condition (based on a FOC) under truth-telling.

- Government problem is then:

$$\max_{c(n),u_n,z_n} \int_0^\infty G(u_n) f(n) dn \qquad \text{subject to}$$

$$\frac{du_n}{dn} = \frac{\ell_n v'(\ell_n)}{n} \qquad \text{(incentive compatibility)}$$

$$\int_0^\infty c(n) f(n) dn \leq \int_0^\infty z_n f(n) dn - E \qquad \text{(resource constraint)}$$

- Much nicer! But how do we solve this?

Mirrlees: Optimal Control

- One of Mirrlee's most interesting insights was that we could take this model and solve it with the machinery of optimal control theory, a mathematical toolkit that is most often used to solve continuous-time dynamic optimization problems.
- The Mirrlees model is not dynamic in the conventional sense - there isn't time! Nonetheless, we can use optimal control theory and regard the type space (n) as the continuous element.
- Key tool: **Hamiltonians**. If you haven't used them before, solving optimal control problems with Hamiltonians is very similar to using Lagrangians for constrained optimizations.

Mirrlees: Hamiltonian

- Define a Hamiltonian function $\mathcal{H}$ incorporating the objective function and constraints:

$$\mathcal{H} = \left[G\left(u_n\right) + p\left(n\ell_n - u_n - v(\ell_n)\right)\right] f(n) + \phi(n)\frac{\ell_n}{n}v'\left(\ell_n\right)$$

  where $\phi(n)$ is the multiplier on the type n incentive constraint and λ is the multiplier on the resource constraint. In optimal control, we call the $\phi(n)$ multiplier (on the type n incentive constraint, i.e. the envelope condition) the costate: note it depends on type n.

- Solving this Hamiltonian means solving out for u_n, ℓ_n, $\phi(n)$, and p. This involves a slightly different approach from a Lagrangians because the costate is a more complicated object than a static constraint. Best illustrated by example.

Mirrlees: Hamiltonian FOCs

- First order necessary conditions for this Hamiltonian:

$$\frac{\partial \mathcal{H}}{\partial \ell_n} = 0 \quad : \quad p \cdot [n - v'(\ell_n] f(n) + \phi(n)/n \cdot [v'(\ell_n) + \ell_n v''(\ell_n)] = 0 \tag{1}$$

$$\frac{\partial \mathcal{H}}{\partial u_n} = -\phi'(n) : \quad \left[G'(u_n) - p\right] f(n) = -\phi'(n) \tag{2}$$

- Also need transversality (or boundary) conditions, $\phi(0) = \phi(\infty) = 0$.
- To solve for costate $\phi(n)$, integrate (2) over $[n, \infty)$, employing transversality condition $\phi(\infty) = 0$ and fundamental theorem of calculus:

$$\phi(n) = \int_n^\infty \left[G'(u_m) - p\right] f(m) dm \tag{3}$$

- To solve for multiplier p, integrate (2) over $[0, \infty)$, and impose both tranversality conditions with the fundamental theorem of calculus to eliminate all ϕ terms:

$$p = \int_0^\infty G'(u_m) f(m) dm \tag{4}$$

Mirrlees: Optimal Tax

- So far, government problem has been in terms of choosing allocations... so how does the tax schedule pop out? Household FOC $n - v'(\ell_n) = nT'(z_n)$ and ϵ.
- Recall (1), the FOC for the Hamiltonian with respect to ℓ_n:

$$p \cdot [n - v'(\ell_n]f(n) + \phi(n)/n \cdot [v'(\ell_n) + \ell_n v''(\ell_n)] = 0$$

- Notice that we can express the last term in terms of the elasticity ϵ after imposing household FOC:

$$\left[v'(\ell_n) + \ell_n v''(\ell_n)\right]/n = \left[1 - T'(z_n)\right] \cdot \left[1 + 1/\epsilon\right]$$

- FOC becomes:

$$p \cdot [nT'(z_n)]f(n) + \phi(n)\left[1 - T'(z_n)\right] \cdot \left[1 + 1/\epsilon\right] = 0$$

Mirrlees: Optimal Tax

- Almost done! Just need to get rid of multipliers $\phi(n)$ and p by substituting (3) and (4):

$$\int_0^\infty G'(u_m)f(m)dm \cdot [nT'(z_n)]f(n) + \int_n^\infty \left[G'(u_m) - p\right]f(m)dm\left[1 - T'(z_n)\right] \cdot \left[1 + 1/\epsilon\right] = 0$$

- Just algebra now - almost exactly identical to the algebra from Saez models. Define marginal social welfare weight as $g_m = G'(u_m)/p$, divide both sides by p, move T' terms:

$$\frac{T'(z_n)}{1 - T(z_n)} = \left(1 + \frac{1}{\epsilon}\right)\left(\frac{\int_n^\infty (1 - g_m)dF(m)}{nf(n)}\right)$$

- Contrast with similar formula for Saez (recall $\alpha(z) = z \cdot h(z)/(1 - H(z))$):

$$\frac{T'(z_n)}{1 - T(z_n)} = \frac{1}{\epsilon_z}\left(\frac{1 - H(z)}{z \cdot h(z)}\right) \cdot (1 - G(z_n)) \quad \text{with} \quad G(z) = \frac{\int_z^\infty g(s)h(s)ds}{1 - H(z)}$$

Mirrlees: Optimal Tax

- Mirrlees optimal tax $T'(z)$ satisfies:

$$\frac{T'(z_n)}{1 - T(z_n)} = \left(1 + \frac{1}{\epsilon}\right)\left(\frac{\int_n^\infty (1 - g_m)dF(m)}{nf(n)}\right)$$

- What are the important take-aways?

- Review question: Can the optimal tax rate $T'(z)$ be negative for any z? Remember our assumptions about the support of n.

- Review question: Suppose the support of n is bounded: $[0, \bar{n}]$. What is the optimal marginal tax rate at the top of the income/ability distribution, $T'(z_{\bar{n}})$?

Mirrlees vs. Saez Taxation

- The optimal tax formula in the Mirrlees model looks a little different from the Saez formula, but they are in fact equivalent if we can map between Mirrleesian types n to Saez incomes z.
- Easy to do (algebra in lecture slides) by considering a linearized budget constraint (local approach).
- Trade-offs in modeling: Mirrlees model is somewhat *richer* in that it requires being more explicit about model primitives, household optimization, and mechanism design.
- Saez approach not as structural; delivers a similar formula, implicitly requires elasticity of taxable income be well-behaved.