Economics 2450A: Public Economics and Fiscal Policy I

Section 2: Optimal Income Taxation I

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Outline

1. Motivation
2. A Simple Optimal Income Tax Model
3. Interlude: Social Welfare Functions
4. Optimal Linear Income Taxation

Motivation

Motivation

- Optimal income taxation is really a highlight of this course. The models are elegant; there are powerful, intuitive trade-offs at play (equity vs. efficiency); the extensions are satisfying.
- Lots of interesting history here - modern optimal tax theory goes back at least to Edgeworth (1897).
- Our lecture/section discussion of optimal income taxation **closely follows** Piketty and Saez (2013), Handbook of Public Economics, Vol. 5. This is a very clearly-written handbook chapter reviewing optimal labor/income taxation, and it will serve as a nice compliment to the section/lecture materials. **Please refer to this if the slides are confusing, it's super helpful!**

Simplest Optimal Income Tax Model

Simplest Optimal Income Tax Model

- We'll start our modeling of optimal taxation with a simple case, in which household income is exogenous and therefore households do not respond to taxation.
- The objective of the government is to maximize what we will call a utilitarian social welfare function, which is simply the sum or integral of utilities across agents (more precisely defined in a few slides).
- These are very strong assumption we will want to relax. But even this very simple model has a powerful insight we can use as a benchmark for more complicated models.

Household Problem

- There is a unit mass of households which are heterogeneous with respect to income z. The distribution of income z across households has density $h(z)$ and support $[0, \infty)$.

- Households derive utility from consumption. Their budget constraint is $c = z - T(z)$.

- Since income z and taxes/transfers $T(z)$ are exogenous, the household utility maximization problem is trivial. The household consumes all their income per their budget constraint, and household i's utility is $u(c_i) = u(z_i - T(z_i))$, where we assume u is concave increasing and differentiable in c.

- The household's problem in this model does exhibit an interesting trade-off; their optimal behavior is simply characterized by the budget constraint!

Government Problem

- Government chooses tax schedule $T(z)$ to maximize social welfare, subject to the constraint that tax revenue must exceed exogenous revenue requirement E:

$$\max_{T(z)} \int_0^\infty u\big(z - T(z)\big)dh(z) \quad \text{s.t.} \quad \int_0^\infty T(z)dh(z) \geq E$$

- Since income z is exogenous and the government can choose $T(z)$ for each z, this is a pointwise maximization problem: can fix z and solve for $T(z)$.

- Form Lagrangian function corresponding to problem, fixing z:

$$\mathcal{L}(T(z), \lambda) = u\big(z - T(z)\big)h(z) + \lambda\Big[T(z)h(z) - E\Big]$$

Government Problem

- First-order condition is simply:

$$u'\big(z - T(z)\big) = \lambda \quad \text{for all } z$$

  where λ is the Lagrange multiplier for the constraint: a **constant**!

- Implies $c = \bar{z} - E$ where $\bar{z} = \int_0^\infty zh(z)dz$

Review Questions (solutions)

- Review question #1: How does E impact λ? What is the intuition?
  Answer: Suppose E goes down. Then c goes up (previous slide: $c = \bar{z} - E$), and if utility is increasing and concave in c, this means marginal utility $u'(c)$ goes down. And since $u'(\cdot) = \lambda$ for everyone, this means λ goes down. So E and λ move together. Intuition: λ is the value of reducing E by a dollar on social welfare. When E goes down, λ is lower because it increases the amount households are consuming, so their marginal utility goes down, which means that the value of relaxing the constraint must fall.

Review Questions (solutions)

- Review question #2: Describe the government's optimal tax policy characterized by the FOC above in words. What is the government doing? Why?
  Answer: The government's optimal tax schedule $T(z)$ is such that marginal utility is constant for all individuals (or incomes), and therefore post-tax income is equalized across people. This result relies on the fact that our utility function is concave and the government is simply maximizing an integral over every household i's utility. Intuitively, if marginal utility was not equalized across people for some tax schedule $T(z)$, the government could increase social welfare by increasing the tax paid by a person with low marginal utility ($z - T(z)$ high) and transferring that money to a person with high marginal utility ($z - T(z)$ low).

Review Questions (solutions)

- Review question #3: What key assumptions does this (strong) result rely on?
  Answer: This model makes three very strong assumptions:
  1. Utilitarian social welfare function (integral of everyone's utilities, no weights - more on next few slides)
  2. Diminishing marginal utility
  3. No behavioral responses to taxes: exogenous z

Social Welfare Functions

Motivation

- Our first optimal tax problem made two very strong assumptions we will relax:
    1. No behavioral responses to taxation: income fixed, consumption and utility only respond mechanically to taxes.
    2. Utilitarian social welfare function: implicit weighting across individuals i.

- Behavioral responses introduce an efficiency angle to the optimal tax problem.

- Generalizing the social welfare function allows us to capture arbitrary preferences for redistribution.

Social Welfare Functions

- What should the government be maximizing? A deeper question than we have time for in this course!

- We will proceed by assuming the government is maximizing a *welfarist* social welfare function, which means that social welfare is a function of all household's utility:

$$\text{Generalized SWF} = \int G\big(u^i(c,z)\big)di$$

  where $G(\cdot)$ is a concave increasing, differentiable function of utility.

- The *marginal social welfare weight* for individual i is defined as:

$$g_i = \frac{G'(u^i)u^i_c}{\lambda}$$

- Interpretation: g_i measures the marginal value (in terms of social welfare) of the gov't giving a dollar to person i.

Common SWFs

- We will see a few different kinds of social welfare functions. Here are some common ones:
  1. A utilitarian social welfare function maximizes aggregate utility: $G(u^i) = u^i$, so $SWF = \int u^i di$.
  2. A Rawlsian (or maxi-min) social welfare function maximizes the minimal utility attained by a person (make the worst-off person as good as possible). Can think of Rawlsian SWF as an $G(\cdot)$ arbitrarily concave near origin; for instance, $G(u) = u^\sigma$ for very small $\sigma > 0$. Implies $SWF = \min u^i$.
  3. A Generalized SWF with Pareto weights can be written as $SWF = \int \mu_i u^i di$, where μ_i are exogenous parameters called *Pareto weights*,

- Review question: Suppose we have a generalized SWF with Pareto weights μ_i. What Pareto weights correspond to utilitarian social welfare? What about Rawlsian social welfare? Answer: Utilitarian case: $\mu_i = 1$ for all i. Should be easy to see. Rawlsian case: $\mu_i = 0$ for all but the lowest earner.

Optimal Linear Income Taxation

Household

- Household problem looks a lot like our consumption-labor supply model from last week (but with choice of taxable income, z, rather than labor supply ℓ).

- There is population of households indexed on the unit interval (normalized mass to 1). Each household chooses consumption c and taxable income z to maximize utility subject to a budget constraint. For simplicity, assume heterogeneity is baked into the utility function. Represent household i's utility function as $u^i(c, z)$, and household UMP is:

$$\max_{c,z} u^i(c,z) \quad \text{s.t.} \quad c = (1-\tau)z + R$$

  where τ is an (exogenous) linear tax rate, R is demogrant funded by taxes taken as given.

- Note that the household takes both τ and R as given! Each household is a 'drop in the bucket' for overall tax revenue, so one household working more does not increase R.

Taxable Income and Tax Revenue

- First-order condition for households: $(1-\tau)u_c^i + u_z^i = 0$

- Solution to UMP yields schedule for taxable income, $z^i(1-\tau, R)$.

  Why write as a function of net of tax rate, $1-\tau$, and not just τ? Convention... sorry!

- Aggregate individual labor supply choices by integrating over i to define aggregate taxable income as a function of the net of tax rate:

$$Z(1-\tau) \equiv \int_i z^i(1-\tau, R)di$$

- Aggregate tax revenue, which is rebated lump-sum to consumers as the demogrant, is simply $R(\tau) = \tau \times Z(1-\tau)$: the constant linear tax rate times aggregate taxable income.

- Note that $R(0) = 0$ and $R(1) = 0$. Embodies the "Laffer curve": revenue maximized in the interior, $\tau \in (0, 1)$.

Elasticity of Taxable Income

- Note that the household's preferences are over consumption and taxable income z. So rather than a labor supply elasticity, there is a taxable income elasticity in this model.

- The taxable income elasticity is so important we'll give it a nice letter, e.

- Convention to define it in terms of the net-of-tax rate $1-\tau$ so that it is generally non-negative:

$$e \equiv \frac{1-\tau}{Z}\frac{\partial Z}{\partial(1-\tau)}$$

- e is the most important elasticity in this course! We will soon call e a *sufficient statistic* for optimal income taxation; the optimal tax rate depends critically on it.

- *"If the net-of-tax rate increases by 1%, how much (in %) does total income change?"*

Review: Revenue-Maximizing Tax Rate

- Review question: Derive the revenue-maximizing tax rate as a function of the elasticity of taxable income e.

- Answer: Tax revenue is $R(\tau) = \tau \cdot Z$. The first-order condition is:

$$
\begin{aligned}
0 &= Z + \tau \frac{\partial Z}{\partial \tau} \\
&= Z - \tau \frac{\partial Z}{\partial (1-\tau)} \\
&= Z - \tau \left[e \frac{Z}{1-\tau} \right] \\
\implies 1 &= \frac{\tau}{1-\tau} e \implies \tau = \frac{1}{1+e}
\end{aligned}
$$

where the first line follows from the product rule, the second line follows from the fact that $\partial y / \partial (1-x) = -\partial y / \partial x$ (useful to memorize), the third line follows from the definition of e, and the last line follows by algebra.

Optimal Policy

- Government chooses τ to maximize the social welfare function, taking as given optimal behavior from household:

$$\max_{\tau} \int_i G\left[u^i\left((1-\tau)z^i + \tau Z(1-\tau), z^i\right)\right] di$$

- For simplicity, have abstracted from revenue requirement E from previous model. Easy to incorporate, but without a revenue requirement, we don't even need a Lagrangian (unconstrained optimization).

- Critical: z^i is the household's optimal choice of taxable income from previous slide. So the government is maximizing a function that *depends on an optimized choice variable*.

- What theorem do you think comes next?

Optimal Policy

- Taking first-order conditions:

$$\frac{\partial}{\partial\tau}\left[\int_i G\Big(u^i((1-\tau)z^i+\tau Z(1-\tau),z^i)\Big)di\right]=0 \qquad \text{(first order condition)}$$

$$\int_i \frac{\partial}{\partial\tau}\left[G\Big(u^i((1-\tau)z^i+\tau Z(1-\tau),z^i)\Big)\right]di=0 \qquad \text{(pass } \tfrac{\partial}{\partial\tau} \text{ inside integral)}$$

$$\int_i G'(u^i)\cdot u^i_c\cdot\left[-z^i+Z-\tau\frac{\partial Z}{\partial(1-\tau)}\right]di=0 \qquad \text{(differentiation + envelope thm.)}$$

- At this point, the lecture slides state this implies the optimal linear tax rate τ is:

$$\tau=\frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g}\equiv\frac{\int g_i z^i di}{Z\int g_i di} \quad \text{and} \quad g_i\equiv G'(u^i)u^i_c \quad \text{and} \quad e\equiv\frac{\partial Z}{\partial(1-\tau)}\frac{1-\tau}{Z}$$

... but it will be really useful to walk through the algebra together and make sure that we can derive it.

Optimal Policy: Envelope Theorem

- Back up a second: it is useful to make sure we all understand how we went from the second line to the third line on the previous slide (the envelope theorem).

- Explicitly differentiating from the second line of the previous slide:

$$0 = \int_i \frac{\partial}{\partial \tau}\left[G\Big(u^i\big((1-\tau)z^i + \tau Z(1-\tau), z^i\big)\Big)\right] di$$
$$= \int_i \left[G'(u^i)\cdot\left[u^i_c\left[(1-\tau)\frac{\partial z^i}{\partial \tau} - z^i + Z - \tau\frac{\partial Z}{\partial(1-\tau)}\right] + u^i_z\frac{\partial z^i}{\partial \tau}\right]\right] di$$

- Critical: first-order condition for household is $u^i_c(1-\tau) + u^i_z = 0$. See the red terms above: these cancel when we impose the household FOC, yielding the expression from the previous slide!

Optimal Policy: Deriving τ

- Now let's finish solving for the optimal tax rate. Starting from line three of slide 18:

$$\int_i G'(u^i) \cdot u^i_c \cdot \left[-z^i + Z - \tau \frac{\partial Z}{\partial(1-\tau)} \right] di = 0$$

- Substitute in $g_i = G'(u^i)\dot{u}^i_c$ and e:

$$\int_i g_i \cdot \left[-z^i + Z - \tau e \frac{Z}{1-\tau} \right] di = 0$$

- Distribute integral over addition/subtraction (integral is a linear operator):

$$-\int_i g_i z^i di + \int_i g_i Z di - \int_i g_i \tau e \frac{Z}{1-\tau} di = 0$$

Optimal Policy: Deriving τ

- Rearrange terms and pull some constants out of the integrals:

$$-\int_i g_i z^i di + Z \int_i g_i di = \frac{\tau}{1-\tau} eZ \int_i g_i di$$

- Divide both sides by $Z \int_i g_i di$, define $\bar{g} = \frac{\int g_i z^i di}{Z \int g_i di}$:

$$1 - \bar{g} = \frac{\tau}{1-\tau} e$$

- A little bit of algebra to solve for τ finally yields:

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e}$$

Optimal Policy: Review Questions

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g} \equiv \frac{\int g_i z^i di}{Z \int g_i di} \quad \text{and} \quad g_i \equiv G'(u^i)u_c^i \quad \text{and} \quad e \equiv \frac{\partial Z}{\partial(1-\tau)}\frac{1-\tau}{Z}$$

- Review question #1: What is e? How does the optimal tax rate τ depend on e? Why? Answer: e is the elasticity of taxable income; it tells us how much aggregate taxable income Z changes as the net of tax rate $1-\tau$ changes. Eyeballing our expression for τ, since e enters positively in the denominator it is evident that τ is decreasing in e. Intuitively, e captures (aggregate) behavioral responses to taxes: if e is higher, there is more of a 'behavioral response'. Tax rates will be higher in the absence of behavioral responses.

Optimal Policy: Review Questions

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g} \equiv \frac{\int g_i z^i di}{Z \int g_i di} \quad \text{and} \quad g_i \equiv G'(u^i)u_c^i \quad \text{and} \quad e \equiv \frac{\partial Z}{\partial(1-\tau)}\frac{1-\tau}{Z}$$

- Review question #2: What is $\bar{g}$? When is $\bar{g}$ large? How does optimal tax formula depend on $\bar{g}$? Why?
  Answer: This is a hard one! At some level, we needed to define $\bar{g}$ to obtain a simple expression for τ: $\bar{g}$ takes care of the integrals. But of course there is a nice interpretation too. The name is meant to be suggestive since we typically use 'bars' to denote averages of some sort. One interpretation of $\bar{g}$ is that it is an income-weighted average of the generalized marginal social welfare weights g_i. The numerator is essentially an income-weighted sum of g_i's, and the denominator normalizes it so that it can be interpreted as an average.

Optimal Policy: Review Questions

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g} \equiv \frac{\int g_i z^i di}{Z \int g_i di} \quad \text{and} \quad g_i \equiv G'(u^i) u^i_c \quad \text{and} \quad e \equiv \frac{\partial Z}{\partial (1-\tau)} \frac{1-\tau}{Z}$$

- Review question #3: How can we interpret this formula as an equity-efficiency trade-off?
  Answer: The object $\bar{g}$ captures the social planner's preferences for redistribution (equity) and the endogenous distribution of taxable income. It is endogenous because it depends not only on $G(\cdot)$, but also on z^i and u^i_c (which are endogenously determined by each household). So $\bar{g}$ is not a parameter, but an endogenous object that captures the value of redistribution in terms of social welfare, which depends on the income distribution and on the social welfare function, The elasticity of taxable income, e, captures behavioral responses to taxation and therefore the 'efficiency' side of the trade-off.

Next Week

- Next week, we'll discuss optimal nonlinear income taxation, building directly off this week's results!
- You'll start to see patterns and big-picture lessons for optimal taxation.
- Make sure you understand the derivation and interpretation of the optimal linear tax rate! You should be able to derive the optimal τ yourself, given the household and government problems.