Economics 2450A: Public Economics and Fiscal Policy I

Section 1: Microeconomics Review

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Fall 2022

Outline

1. Section Logistics
2. Microeconomics review I: Static consumption model
   - Utility maximization problem, Marshallian demand
   - Expenditure minimization problem, Hicksian demand
   - Uncompensated and compensated price elasticities
   - The Slutsky equation
3. Microeconomics review II: Consumption-labor supply model
   - Uncompensated labor supply elasticity
   - Compensated labor supply elasticity
   - Welfare impact of taxes
   - Looking ahead to next week

Section Logistics

Section Logistics

Teaching fellow: Michael Droste (mdroste@fas.harvard.edu)

- G5 in economics; I study macro and public.

Sections: Held twice each week (same content both days)

- Wednesday: 4:30-5:45pm, Sever Hall 210 (location subject to change in first 3 weeks)
- Thursday: 12:00-1:15pm, Sever Hall 306 (location subject to change in first 3 weeks)

Times are tentative: please see my 9/7 announcement on Canvas!

Office hours: Held twice per week in Littauer basement common area:

- Wednesday: 2:45-4:00pm (after lecture)
- Thursday: 1:30pm-2:45pm

Please feel free to reach out to me with *any* questions about the course - I am happy to talk with you outside section/OHs (e-mail, in-person, or on Zoom) whenever you like.

Sections

- Section is a place where we can work through the more complex elements that appear in lecture together, with a focus on the math and economic intuition.

- Section notes and slides will be posted on Wednesdays. The section notes will contain 'practice problems' that we will review in section. Slides will be updated with solutions on Thursday.

- We have room in section to discuss topics of interest to you personally. Please let me know in advance if you would like us to devote more time to an element of the lectures or problem sets.

Math and Micro Review Handout

- I have posted mathematics and microeconomics review notes on Canvas. This is a completely optional document that reviews the key mathematical tools and elements of consumer theory you will see in this course.
- Please review this document if you're feeling rusty. **It is totally OK to be rusty!** You will have plenty of time to practice this semester.
- Pay particular attention to the envelope theorem. The envelope theorem will be your best friend in the public economics sequence. It makes life easier. We will see applications all the time.

Today's Goal

- Today's plan is to review static partial equilibrium consumption models. This content should largely be familiar to you from prior coursework.

- Model #1: N-good consumption model with exogenous prices and income.
  - Household problem: choose how to allocate income across goods.
  - Useful to refresh our consumer theory

- Model #2: consumption-labor supply model.
  - Household problem: choose how much to consume and how much to work
  - Study implications of an exogenous tax
  - Basis for next week's discussion of optimal income taxation

Microeconomics Review I: Consumption

Static Consumption Model: Motivation

- We'll begin with a very simple partial equilibrium model in which a representative household chooses consumption of N goods, with exogenous income and prices.

- Household has preferences over N goods, indexed by i. These preferences are characterized by the utility function $u(x_1, ..., x_N)$, where x_i denotes the quantity consumed of good i. We assume utility function is differentiable, increasing, and concave in each good.

- Household chooses consumption bundle $\mathbf{x} = (x_1, ..., x_N)$ that maximizes utility subject to a budget constraint, given exogenous prices p_i and income z.

Utility Maximization Problem

- Utility maximization problem for the household:

$$\max_{x_1,\ldots,x_N} \ u(x_1,\ldots,x_N) \quad \text{subject to} \quad \sum_{i=1}^{N} p_i x_i \leq z$$

- More concisely in vector notation:

$$\max_{\mathbf{x}} \ u(\mathbf{x}) \quad \text{subject to} \quad \mathbf{p}\cdot\mathbf{x} \leq z$$

- Assume utility function is well-behaved: smooth and quasiconcave. Walras' law implies budget constraint binds at an optimum: $\sum_{i=1}^{N} p_i x_i = z$.

Lagrangian and FOCs

- Form the Lagrangian corresponding to the utility maximization problem (UMP):

$$\mathcal{L} = u(x_1, \ldots, x_N) + \lambda \left(w - \sum_{i=1}^{N} p_i x_i \right)$$

- Solve for the optimal choices of x_i by taking first-order conditions (FOC):

$$u_i\left(\mathbf{x}^*\right) - \lambda p_i = 0 \quad \forall i$$

- Then for any two goods i and j, their FOCs imply:

$$\text{marginal rate of substitution} \equiv \frac{u_i\left(\mathbf{x}\right)}{u_j\left(\mathbf{x}\right)} = \frac{p_i}{p_j}$$

- Optimal consumption behavior implies that marginal utility per dollar spent on each good will be equated across all N goods.

Review Problems

- Review problem 1: What is the interpretation of the Lagrange multiplier λ?
  - Solution: The Lagrange multiplier for a constrained maximization problem always answers the question, "What is the marginal value (in terms of the objective/value function) of relaxing the constraint?". In this case, it tells us what the marginal utility out of wealth is.

- Review problem 2: Let p be the price for a particular good. Please interpret: $\frac{du/dp}{\lambda}$.
  - Solution: du/dp is marginal utility from a change in p; λ is the marginal utility of wealth. So $(du/dp)/\lambda$ indicates willingness to pay for a price change dp: how much (in utility terms) a price change dp worth relative to a marginal increase in income.

Review Problems

- Review problem 3: Could we measure $\frac{du/dp}{\lambda}$? Describe a setting in which we could (with the appropriate estimator and source of variation, i.e. an experiment or policy change that can be used with quasi-experimental methods).
  - Solution: We could estimate this with an experiment eliciting consumer's WTP, randomly varying prices p. WTP is hard to measure in practice: 2450B will cover how to measure this quantity using the envelope theorem (Hendren and Sprung-Keyser's marginal value of public funds).

Marshallian Demand and Uncompensated Demand Elasticity

- The solution to the utility maximization problem can be expressed as a (vector-valued) function $\mathbf{x}(\mathbf{p}, w)$ of the price vector and the consumer's income.

- We call $\mathbf{x}(\mathbf{p}, w)$ the Marshallian demand function. It is a vector-valued function; the ith element $x_i(\mathbf{p}, w)$ gives the demand for good i as the function of the price vector $\mathbf{p}$ and the wage w.

- The *uncompensated* price elasticity of demand for good i is:

$$\varepsilon^u_{i,p_k} = \frac{\partial x_i(\mathbf{p}, w)}{\partial p_k} \frac{p_k}{x_i(\mathbf{p}, w)}$$

- The percentage change in the consumption of good i when p_k increases.

Indirect Utility

- **Indirect utility:** the utility that the consumer realizes when consuming the optimal bundle $\mathbf{x}(\mathbf{p}, w)$.

- It can be obtained by plugging Marshallian demand into the utility function:

$$v(\mathbf{p}, w) = u(\mathbf{x}(\mathbf{p}, w))$$

- Indirect utility, usually denoted as v or V, is the value function corresponding to the utility maximization problem.

Expenditure Minimization Problem

- Dual of utility maximization problem is the expenditure-minimization problem:

$$\min_{x_1,\ldots,x_N} \quad \sum_{i=1}^{N} p_i x_i \quad \text{subject to} \quad u(x_1,\ldots,x_N) \geq \bar{u}$$

- The solution to the EMP is the Hicksian (or compensated) demand function, which is often denoted either as $\mathbf{h}(\mathbf{p},\bar{u})$ or as $\mathbf{x}^c(\mathbf{p},\bar{u})$.

- The expenditure function is the value function corresponding to the expenditure minimization problem, denoted $e(\mathbf{p},\bar{u})$. As before, the expenditure function can be obtained by plugging in h into the EMP.

Hicksian Demand: Interpretation

- Hicksian demand isolates the pure *substitution effect* in response to a price change.
- **Why "compensated"?** The EMP holds utility fixed, so you can suppose that after a price change, consumer's income adjuss to maintain the same utility level realized prior to the price change.
    - Under prices $\mathbf{p}$, the consumer demands $\mathbf{x}$ such that $\mathbf{p} \cdot \mathbf{x} = w$.
    - When prices become $\mathbf{p}'$, consumer demands $\mathbf{x}'$ such that $u(\mathbf{x}) = u(\mathbf{x}')$ and $\mathbf{p}' \cdot \mathbf{x}' = w'$
    - The consumer needs $\Delta w = w' - w$ to be as well off as she was before.
    - This avoids any reduction in the consumer's purchasing power: they are "made whole"
- The *compensated elasticity* corresponds to Hicksian demand:

$$\varepsilon^{c}_{i,p_k} \equiv \frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_k} \frac{p_k}{h_i(\mathbf{p}, \bar{u})}$$

Hicksian Demand: Review

- Review problem: In what sense does the setup of the expenditure minimization problem (EMP) guarantee that the consumer is “compensated” or “made whole” by the price change?
  - Solution: Income is not in the expenditure minimization problem at all! Instead, the expenditure minimization problem holds utility fixed. In order for utility to remain fixed when a price changes, income (in the UMP sense) must be implicitly adjusting. In other words, the expenditure minimization problem is set up so that the consumer can spend as much income as they want to minimize expenditures so as to attain an exogenous level of utility. Because there is no explicit income in the EMP constraint or objective function, there are no income effects.

Utility Maximization-Expenditure Minimization Duality

- The household's utility-maximization problem (UMP) and expenditure-minimization problem (EMP) are *duals*. Their solutions answer related questions:
  1. If $\mathbf{x}^*$ is optimal in the UMP when wealth is w, then $\mathbf{x}^*$ is optimal in the EMP when $\bar{u} = u(\mathbf{x}^*)$. Moreover, $e(\mathbf{p}, \bar{u}) = w$.
  2. If $\mathbf{x}^*$ is optimal in the EMP when $\bar{u}$ is the required level of utility, then $\mathbf{x}^*$ is optimal in the UMP when $w = \mathbf{p} \cdot \mathbf{x}^*$. Moreover, $\bar{u} = u(\mathbf{x}^*)$.

- These are rich statements. Mathematically, it's often useful to express this duality as:

$$\mathbf{x}(\mathbf{p}, e(\mathbf{p}, \bar{u})) = \mathbf{h}(\mathbf{p}, \bar{u}) \quad \text{where} \quad u(\mathbf{x}(\mathbf{p}, w)) = \bar{u} \quad \text{and} \quad e(\mathbf{p}, \bar{u}) = w$$

- Intuitively: Marshallian and Hicksian demands *coincide in equilibrium*.

The Slutsky Equation

- Totally differentiating the duality relationship $x_i(\mathbf{p}, e(\mathbf{p}, \bar{u})) = h_i(\mathbf{p}, \bar{u})$ with respect to p_k yields the Slutsky equation for any two goods i and k:

$$\underbrace{\frac{\partial x_i(\mathbf{p}, w)}{\partial p_k}}_{\text{uncompensated change}} = \underbrace{\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_k}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_i(\mathbf{p}, w)}{\partial w} x_k(\mathbf{p}, w)}_{\text{income effect}}$$

- The income effect is the product of two terms:
  1. $\frac{\partial x_i(\mathbf{p}, w)}{\partial w}$ is the response of the demand of good i to a change in wealth,
  2. $x_k(\mathbf{p}, w)$ is the mechanical effect of an increase in p_k on the agent's purchasing power.

- I have omitted the algebra here because we have bigger fish to fry, but in case you're curious, it's in the math/micro review document posted on Canvas.

Microeconomics Review II:
Labor Supply

Static Labor Supply - Setup

- Next: consider a model where income is endogenous.

- Representative household has preferences over consumption c and labor supply ℓ, represented by smooth and quasiconcave utility function $u(c, \ell)$.

- Trade-off: household can work for more labor income $w\ell$, which they can spend on a consumption good for more utility. But working sucks (disutility from more labor supply). The optimal choice of (c, ℓ) trades off the disutility from work against the utility from consumption.

Utility Maximization Problem

- Household chooses consumption c and labor supply ℓ to maximize utility subject to a budget constraint:

$$\max_{c,\ell} u(c,\ell) \quad \text{subject to} \quad c = w\ell + y$$

  where w denotes the exogenous wage rate, and y denotes exogenous non-labor income.

- Solving the Lagrangian yields an optimality condition that characterizes the trade-off between consumption and leisure:

$$-\frac{u_\ell}{u_c} = w$$

Optimality Condition from Perturbation

- Next week, we will begin to see optimal policy problems, where the government's choice of tax rate solves an appropriate optimization problem, taking as given the consumer's behavior.

- It is often useful to solve these problems with a perturbation or variational argument, which you likely saw in first year theory courses. We can apply the same approach to derive the solution to utility maximization problems without explicitly solving Lagrangians.

- The following review problem is intended to be a little bit of practice for next week.

- Review Problem: Derive the optimality condition (and provide intuition) using a perturbation approach / variational argument.

Review Problem (Solution)

- We start by considering a feasible allocation (c, ℓ) that satisfies the budget constraint.
- Now consider a small perturbation of this allocation that is also feasible. We can perturb either c or ℓ either up or down (end result is the same). Let's suppose we 'perturb' ℓ by considering a feasible allocation with $\ell + \epsilon$ for some very small ϵ.
- Under this new allocation, you work ϵ units more, and therefore earn $\epsilon \times w$ more dollars. From the budget constraint, this yields $u_c \times w \times \epsilon$ units more utility from consumption.
- This perturbation also induces *dis*utility from increased labor supply. In particular, working more reduces utility by $-\epsilon \times u_\ell$.
- If the original allocation (c, ℓ) is optimal, then the gain in utility from consumption will exactly offset the loss in utility from labor supply. This implies $-\epsilon \times u_\ell = \epsilon \times w \times u_c$. Canceling ϵ from both sides yields the desired result.

Uncompensated Labor Supply Elasticity

- Totally differentiating the optimality condition wrt w we get:

$$\frac{\partial \ell}{\partial w} = -\frac{u_c + \ell\left(u_{\ell c} + w u_{cc}\right)}{w^2 u_{cc} + 2w u_{\ell c} + u_{\ell\ell}}$$

- The denominator is the SOC and negative. This implies:

$$\frac{\partial \ell}{\partial w} \propto \underbrace{u_c}_{\text{substitution effect}} + \underbrace{\ell\left(u_{\ell c} + w u_{cc}\right)}_{\text{income effect}}$$

- *Substitution effect:* depends on the marginal utility of consumption (substitute leisure with labor to consume more)

- *Income effect:* 3 components
  - ℓ: mechanical effect on endowment of a change in w
  - $u_{\ell c}$: captures how the increase in consumption affects the marginal disutility from working
  - u_{cc}: faster decreasing marginal returns from consumption lead to a stronger income effect

Example: Constant Uncompensated Labor Elasticity

- Suppose the utility function takes the isoelastic form:

$$u(c, \ell) = c - \frac{\ell^{1+\frac{1}{\varepsilon}}}{1 + \frac{1}{\varepsilon}}$$

- This is a quasi-linear utility function: no income effect. The optimality condition is:

$$\ell^{\frac{1}{\varepsilon}} = w$$

- Taking logs we get:

$$\frac{1}{\varepsilon} \log \ell = \log w$$

- Since $\varepsilon^{u}_{\ell,w} = \partial \log \ell / \partial \log w$ (why?) we can write:

$$\varepsilon^{u}_{\ell,w} = \partial \log \ell / \partial \log w = \varepsilon$$

- Therefore, we have a *constant elasticity of labor supply*.

Income Effect

- What is the *compensated* labor supply response? We can use the Slutsky equation:

$$\frac{\partial \ell}{\partial w} = \frac{\partial \ell^c}{\partial w} + \frac{\partial \ell}{\partial y}\ell$$

- We already found the left-hand side; just need $\partial \ell / \partial y$.

- Totally differentiating the optimality condition with respect to y, we get:

$$\frac{\partial \ell}{\partial y} = -\frac{u_{\ell c} + w u_{cc}}{w^2 u_{cc} + 2w u_{\ell c} + u_{\ell \ell}}$$

- Solving for $\frac{\partial \ell^c}{\partial w}$:

$$\frac{\partial \ell^c}{\partial w} = -\frac{u_c}{w^2 u_{cc} + 2w u_{\ell c} + u_{\ell \ell}}$$

Envelope Theorem

- The envelope theorem often comes up when we take a comparative static of a value function that is being optimized.

- Example: how does a marginal increase in taxes impact consumer welfare? What parameters does this depend on?

- Let's add in an exogenous constant tax rate τ on labor income to our model. For simplicity, we'll assume tax revenue is thrown away.

- The UMP and associated value function can be written as:

$$V(\tau) \equiv \left\{ \max_{c,\ell} \; u(c,\ell) \quad \text{subject to} \quad c = (1-\tau)w\ell + y \right\}$$

Envelope Theorem (continued)

$$V(\tau) \equiv \left\{ \max_{c,\ell} \; u(c, \ell) \quad \text{subject to} \quad c = (1 - \tau)w\ell + y \right\}$$

- Let's how the envelope theorem holds 'from first principles' by taking the comparative static $dV/d\tau$. We will be able to 'prove' the envelope theorem simply from our optimality conditions and single-variable calculus.

- Let's work through this problem on the board.

- A solution will be posted in an updated slide deck on Thursday evening.

Envelope Theorem (solution)

- Since $V(\tau) = \mathcal{L}(c^*, \ell^*, \lambda^*)$ when (c^*, ℓ^*) solve the UMP, we can write $dV/d\tau$ as:

$$\begin{aligned}
\frac{dV}{d\tau} &= \frac{d\mathcal{L}}{d\tau} \\
&= u_c \tfrac{\partial c^*}{\partial \tau} + u_\ell \tfrac{\partial \ell^*}{\partial \tau} + \lambda\left[(1-\tau)w\tfrac{\partial \ell^*}{\partial \tau} - w\ell^* - \tfrac{\partial c^*}{\partial \tau}\right] && \text{(differentiating } \mathcal{L}\text{)} \\
&= \lambda^* \tfrac{\partial c^*}{\partial \tau} - \lambda^*(1-\tau)w\tfrac{\partial \ell^*}{\partial \tau} + \lambda^*\left[(1-\tau)w\tfrac{\partial \ell^*}{\partial \tau} - w\ell^* - \tfrac{\partial c^*}{\partial \tau}\right] && \text{(plug in FOCs)} \\
&= -\lambda w \ell^* && \text{(cancel terms)} \\
&= \frac{\partial \mathcal{L}}{\partial \tau}
\end{aligned}$$

- In a roundabout way, we just 'proved' the envelope theorem (last line): we could have gotten the same result by simply treating the choice variables as fixed!

Envelope Theorem: In Words

- The envelope theorem is powerful! Often arises in models when we want a comparative static of a value function (e.g. indirect utility; social welfare next week) . Also empirically useful: 2450B will examine several empirical applications where the envelope theorem motivates an estimand (i.e. Hendren and Sprung-Keyser 2020).
- My intuition: when an objective function is being optimized, changes in parameters (e.g. τ) have two types of effects: direct effects (through the budget constraint) and indirect effects (through the choice variables). When the change in τ is small (like $d\tau$), the indirect effects are completely overwhelmed by the direct effect. The indirect effect drops out as a consequence of the choice variables being optimized (FOCs holding).

Envelope Theorem: In Math

- The following statement of the envelope theorem applies when the objective function is differentiable with respect to the choice variable. Suppose we choose some variable(s) $\mathbf{x}$ to maximize a function $f(x, \alpha)$ subject to the constraint $g(x, \alpha) = 0$, where α is some interesting parameter. The optimization problem and associated value function can be written as:

$$V(\alpha) = \left\{ \max_{x} \ f(x, \alpha) \quad \text{s.t.} \quad g(x, \alpha) = 0 \right\}$$

- The envelope theorem says that $\frac{dV}{d\alpha} = \frac{\partial \mathcal{L}}{\partial \alpha} = f_\alpha + \lambda^* g_\alpha$, where f_α, g_α denote the partial derivative of f and g with respect to α and λ^* denotes the value of the Lagrangian multiplier at the optimized value of x.
- Notably, this comparative static does not feature terms like f_x or $\frac{\partial x}{\partial \alpha}$ - they drop out as a consequence of optimization.

Envelope Theorem Review

- Review problem #1: Suppose the Earned Income Tax Credit (EITC) is made marginally more generous, and some working parents change their labor supply from 0 hours to 20 hours per week. We are interested in computing the welfare impact of the policy change. Will the envelope theorem hold? Why or why not?
  - Solution: Yes. The envelope theorem can hold for discrete choices. Intuitively, those who are on the margin between working and not working are indifferent in utility terms.

- Review problem #2: Suppose that Congress eliminates the federal income tax. We are interested in computing the welfare impact of the policy change. Will the envelope theorem hold? Why or why not?
  - Solution: No. The envelope theorem only holds exactly for very small policy changes. For large policy changes, we cannot ignore the indirect effects that a parameter has on the choice variables.

Envelope Theorem Review

- Review problem #3: What do these problems suggest about the limitations of the envelope theorem? Are there other limitations you can think of?
  - Solution: The envelope theorem is a useful tool for small policy changes when a value function is being optimized. It does rely on optimality (first-order conditions or indifference conditions for discrete choice); behavioral models, frictions or constraints that interfere with optimality conditions holding can all cause the envelope theorem to fail.

Looking Ahead

- Next week, we'll start to consider models of optimal income taxation that build off this consumption-labor supply model.

- Useful change of variables: define income as $z = w \cdot \ell$, recast consumer problem:

$$\max_{c,z} u(c,z) \quad \text{s.t.} \quad c = z - T(z) + R$$

  where R is exogenous income (e.g. lump-sum rebates from tax revenue), $T(z)$ is a tax schedule (as a function of income)

- Allow for household heterogeneity in various ways; e.g. heterogeneity in the budget constraint (stochastic income or productivity) or in the utility function (heterogeneous preferences). Allows us to explore equity / distributional concerns and equity-efficiency trade-offs.