Economics 1011B
Section 8

Spring 2023

Today's Outline

- Investment
  - Firm Investment witih Adjustment Costs
  - Interpreting the Firm's Multiplier
  - Tobin's Q
- IS-MP
  - Setup
  - Comparative Statics
- Bonus Slides: Tobin's Q

Investment

Model of Firm Investment

- We will now consider a partial-equilibrium model of firm investment.

- Only a little bit different from the firm we saw in the neoclassical growth model. Key differences: assume the firm owns the capital stock and potentially faces 'adjustment costs'.

- Tools for solving this model are the same as before: no new math.

- For simplicity, we will assume the firm's production function only uses capital, no labor. (This assumption does not change the intuition - just simplifies the algebra).

Model of Firm Investment

- Firms produce output using a production function involving previous period capital $F(K_{t-1})$ with positive, diminishing marginal product: $F_K > 0, F_{KK} < 0$.

- Firms own their own capital, which changes over time with the usual law of motion:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- The firm can purchase investment I_t at a per-unit price of p_t^K. If the firm invests, they pay an adjustment cost of $\Phi(I_t, K_{t-1})$. This is a function - it depends on I_t and potentially also K_{t-1}. We typically assume $\Phi(0, \cdot) = 0$, hence, adjustment costs.

- Leading example: quadratic adjustment costs, $\Phi(I_t) = \phi I_t^2$

- Firm wants to maximize expected future discounted dividends. Discount rate is $(1 + r)^{-1}$.

Model of Firm Investment

- Firm's goal is to choose investment/capital to maximize thee present discounted value of profit Firms choose investment to maximize profit, subject to the law of motion for capital:

$$\max_{\{I_t, K_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \frac{F(K_{t-1}) - p_t^K I_t - \Phi(I_t, K_{t-1})}{(1+r)^t} \quad \text{subject to} \quad K_{t+1} = (1-\delta)K_t + I_t \quad \forall t$$

- Key differences from firm problems we've seen before (i..e neoclassical growth):
  #1. Firms own the capital stock: law of motion is a constraint for the firm
  #2. Adjustment cost: Any (nonzero) investment incurs an 'adjustment cost' captured by $\Phi(\cdot, \cdot)$
- Together, these imply firm faces a *dynamic* constrained maximization problem (maximize lifetime discounted profits, not just profits this period) – decisions today impact decisions tomorrow, since investment determines capital available tomorrow.

Model of Firm Investment

- Lagrangian for firm's decision in period t:

$$\mathcal{L} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[F(K_{s-1}) - p_s^K I_s - \Phi(I_s, K_{s-1}) + \lambda_s \left[(1-\delta) K_{s-1} + I_s - K_s \right] \right]$$

- Interpret each term above. Note the constraint is the law of motion for capital. Analogous to consumption-savings: write Lagrangian in terms of c_t and c_{t+1}, even though c_{t+1} determined by (budget) constraint and c_t.

- First-order conditions:

$$\text{For } K_t: \quad \frac{1}{1+r} \left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta) \right] = \lambda_t$$

$$\text{For } I_t: \quad p_t^K + \Phi_{I_t} = \lambda_t$$

Investment: Interpretation of λ_t

- In this model, the Lagrangian multiplier for the firm's problem, λ_t, will be quite important, and have a nice interpretation.

- Recall that generally, the Lagrange multiplier for a constrained optimization problem tells us: what is the "marginal value" of relaxing the constraint by a small amount?

- In this case, λ_t tells us: what is the present discounted value of marginal investment?

- Relationship to asset pricing: market price of additional investment.

Investment: Interpretation of λ_t

- Note that λ_t has a recursive definition. Expanding the λ_{t+1} term T times:

$$
\begin{aligned}
\lambda_t &= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] \\
&= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t}\right] + \frac{1-\delta}{1+r}\lambda_{t+1} \\
&= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t}\right] + \frac{1-\delta}{1+r}\left[\frac{1}{1+r}\left[F_{K_{t+1}} - \Phi_{K_{t+1}} + \lambda_{t+2}(1-\delta)\right]\right] \\
&\vdots \\
&= \frac{1}{1+r}\sum_{s=t}^{T}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right) + \left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1}
\end{aligned}
$$

- This looks boring, but please don't think it's trivial! It is really good practice (for homeworks, or tests) to show that this is true.

Investment: Interpretation of λ_t

- As $T \to \infty$, our expression for λ_t simplifies (under the minor technical condition that the second term goes to zero):

$$\lambda_t = \frac{1}{1+r} \sum_{s=t}^{T} \left(\frac{1-\delta}{1+r} \right)^{s-t} (F_{K_s} - \Phi_{K_s})$$

- Great. Now we can think about what λ means - the PDV of future dividends from a marginal unit of capital.

Investment: Tobin's q

- First-order condition for investment I_t was:

$$p_t^K + \Phi_{I_t} = \lambda_t \iff \frac{\Phi_{I_t}}{p_t^K} = \frac{\lambda_t}{p_t^K} - 1$$

- Define Tobin's q, $q_t = \frac{\lambda_t}{p_t^K}$. Then:

$$\frac{\Phi_{I_t}}{p_t^K} = q_t - 1$$

- If the sign of Φ_{I_t} is the same as the sign of I_t, which we have assumed (and is true for the leading quadratic adjustment cost example), then $I_t > 0 \iff q_t > 1$. In other words, the firm's investment decision boils down to whether q is bigger or smaller than one.

- Try to interpret this economically - it's useful!

IS-MP

Business Cycle Theory

- We are now beginning to cover models of the business cycle.
- Why does the economy go through booms and busts? How can government policy interact with these forces?
- Today we'll work through the "IS-MP" model: a slightly more modern twist on the IS-LM model that every intermediate macro course in the world studies. This model is not based on a 'micro-founded' model with explicitly optimizing agents, and may feel a bit different from what we've seen so far in our models.
- IS-MP is your friend: the simplest model in the course.

IS-MP: Consumption and Investment

- Standard variable definitions: Y is income/output, C is consumption, I is investment, G and T are government expenditures and taxes, and r is the interest rate. G and T are exogenous constants.

- Assume consumption is positively related to disposable income:

$$C = C_0 + C_1 \times (Y - T) \tag{1}$$

  where $0 \leq C_1 \leq 1$ is the marginal propensity to consume out of disposable income.

- Assume investment is positively related to disposable income, negatively related to r:

$$I = I_0 + I_1 \times (Y - T) + I_2(r) \tag{2}$$

  where $I_2(r)$ is a function with $I_2' < 0$ (higher interest rates yield less investment).

IS-MP: IS curve

- Define the "IS" (investment-savings) curve to be the relationship between Y and r such that $Y = C + I + G$, which you can interpret as a goods market clearing condition.

- We can derive the IS curve by taking $Y = C + I + G$, substituting in our consumption and investment functions, and solving for Y as a function of r:

$$\begin{aligned} Y &= C + I + G \\ &= \Big[C_0 + C_1 \times (Y - T)\Big] + \Big[I_0 + I_1 \times (Y - T) + I_2(r)\Big] + G \\ &= \frac{C_0 + I_0 + G - (C_1 + I_1)T}{1 - C_1 - I_1} + \frac{I_2(r)}{1 - C_1 - I_1} \end{aligned}$$

- This is one equation and two unknowns (Y and r). There are a set of (Y, r) that satisfy this equation - not a unique solution.

- What is the relationship between Y and r? What conditions are required for this to be well-defined? Does it make sense?

IS-MP: Monetary Policy (MP) curve

- To solve the model, we need another equation relating Y and r (so we have two equations corresponding to two unknowns).

- The "MP" curve comes from a monetary policy rule: we assume the Fed / central bank is promising to set interest rates according to a rule where interest rates are (weakly) *increasing* in output Y:

$$r = r(Y), \qquad r' > 0$$

- We could parameterize this: for instance, $r = r_0 + r_1 Y$, with $r_1 \geq 0$.

IS-MP: Equilibrium

- The IS and MP curves give us two equations in two unknowns (Y, r).
- Equilibrium in this model is defined as the set of points where both the money market and goods market are in equilibrium. Visually, this is where the IS and MP curves intersect.
- Every point on the IS curve represents a tuple (Y, r) where the goods market is in equilibrium: $Y = C + I + G$.
- Every point on the MP curve represents a tuple (Y, r) where the monetary policy rule is satisfied. Only at the overall equilibrium (where both lines cross) do both occur.

IS-MP: Graphical Interpretation

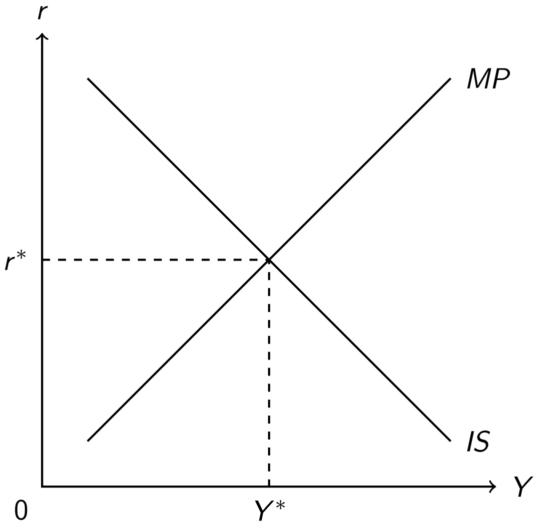
- Comparative statics in this model: what happens to equilibrium output and the interest rate if something changes?

- Huge advantage of this model is that all comparative statics can be interpreted graphically: just need to find out if the IS or MP curve shifts (or both), and in which direction, to find out what happens to output Y and the interest rate r.

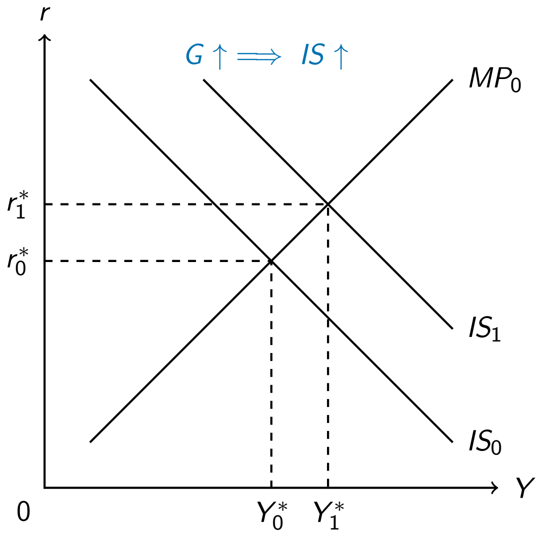
- In my view, the graphical interpretation is both easier and quicker.

- What are the kinds of questions we can answer in this model?
  - What happens if G goes up? What about T?
  - What happens if monetary policy responds more to Y? What if r is higher for any given Y?
  - What happens if household MPC goes up?
  - What happens if there is an exogenous decrease in consumption?

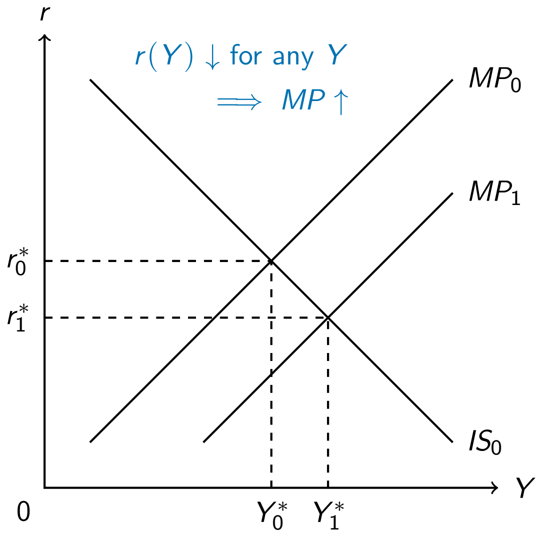
IS-MP: Equilibrium



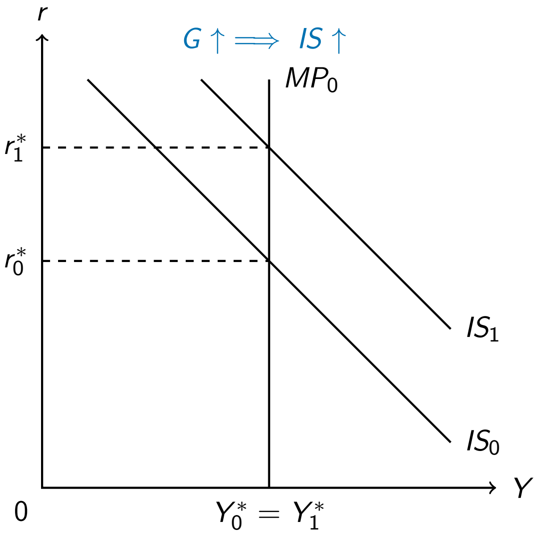
IS-MP: Expansionary Fiscal Policy



IS-MP: Expansionary Monetary Policy



IS-MP: Expansionary Fiscal Policy (Vertical MP)



IS-MP: Geometry of Comparative Statics

- Clearly, the shape of the IS and MP curves matter for the impacts of policy (all the usual geometric lesssons regarding supply/demand curves from ec10 apply!).

- It's worth trying to think through questions like, "What shifts the IS and MP curves?" through a variety of graphical examples.

- Then think through what determines the slopes of the IS and MP curves in this model (the parameters in the consumption/investment/money demand functions - but which ones, and why?).

- You should be able to graphically answer what happens to Y and r when, for instance, G or T change - both graphically and using calculus - to compute an explicit comparative static.

Bonus Slides

Bonus slides: History of Tobin's q

- Tobin did not formalize the idea of Tobin's q - Kaldor (of stylized fact fame) did. He called it v instead.

- But Tobin popularized the idea a decade later than Kaldor, convincing most macroeconomists that it was critical for our understanding of investment. Here's what Tobin had to say about his q: *"One, the numerator, is the market valuation: the going price in the market for exchanging existing assets. The other, the denominator, is the replacement or reproduction cost: the price in the market for newly produced commodities. We believe that this ratio has considerable macroeconomic significance and usefulness, as the nexus between financial markets and markets for goods and services."*

Bonus slides: Measuring q

- How can we measure Tobin's q? It's a "marginal" concept – how productive would an additional unit of investment be? In practice, it's always very difficult to measure 'marginal' things (i.e. marginal product of labor or capital) - no different here.

- With a randomized experiment involving random capital investment across firms/time, maybe we estimate this directly without an economic model: see how much output changes when we randomly assign capital. But such an experiment would be far too costly to run.

- Standard data only allows us to measure "average" q - the market value of an average unit of capital relative to replacement cost.

- With economic theory, we can try to map 'observed' quantities using model-predicted equilibrium relationships to determine Tobin's q from observational data.

- Fun question: What is the relationship between "average" q and "marginal" q (where our q is the marginal q)? In particular, which one is bigger? Why?

Bonus slides: Rational Expectations Revolution

- From the 1940's through the 1970's, macroeconomists were quite happy working with 'top-down', old school Keynesian models like IS-MP.

- These models begin positing relationships between aggregate variables rather than starting from the behavior of individual agents, as we have done elsewhere in this course.

- A huge advantage of this class of models is their analytical tractability - they are easy to solve and to extend with additional features. They are easy to solve, and it is easy to incorporate additional features, almost by fiat.

- But even back then, it was known that this class of models had serious disadvantages: they were tricky to reconcile with microeconomic theory (not being based on microeconomic agents), how expectations were formed (and dynamic aspects of the models more generally) somewhat arbitrary.

Bonus slides: Rational Expectations Revolution

- Macroeconomics shifted rather abruptly away from this class of models in the mid-1970s.

- A growing body of economists began to argue that these models were in many ways undesirable. Among the arguments they raised:
  - The models do not start from individual optimization and are difficult to reconcile with models of microeconomic behavior
  - To the extent that expectations enter in these models (there are IS-MP-type models with dynamics, which we will not go into), they are often totally ad hoc (adaptive expectations) and backward-looking. In the real world, people are forward-looking.
  - The old models did not provide a realistic way to think about the effects of policy changes, since they assumed parameters to be fixed (like MPCs) that are in fact not invariant to policies. The only path forward is to start from modeling individual choices, being careful to start with parameters that can be argued should be invariant to whatever policy is being studied.

Bonus slides: Rational Expectations Revolution

- The timing of the shift away from IS-MP type models was was shaped in large part by the "stagflation" - a period of stagnant economic growth coinciding with substantial inflation - of the United States in the 1970s, which some argued represented a failure of macroeconomic policy that had been shaped by the misguided belief that government could consistently "fool" consumers.
- In particular, the idea of the Lucas critique was often sold in the context of the Phillips curve - the government tried to exploit the Phillips curve as a static equation (like a law of nature); but as they economy 'overheated', the relationship changed.
- In the view of Lucas, this capture dthe need to start from models that captured individuals 'deep' parameters that are invariant to policy changes, like elasticities of demand rather than assuming fixed marginal propesnities to consume.
- In my own (humble) view, stagflation had little to do with US fiscal policy in the 70s (and a lot more to do with oil crises).